University of California, Berkeley
Physics 105 Fall 2000 Section 2 (*Strovink*)

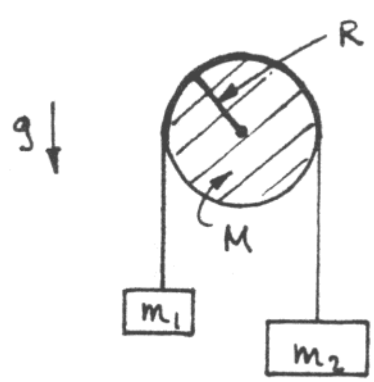
ASSIGNMENT 3

Reading:
105 Notes 4.1-4.6, 5.1-5.3.
Hand & Finch 1.4, 2.1-2.9, 1.10-1.11

1.
Use the calculus of variations to show that the shortest distance between two points in three-dimensional space is a straight line.

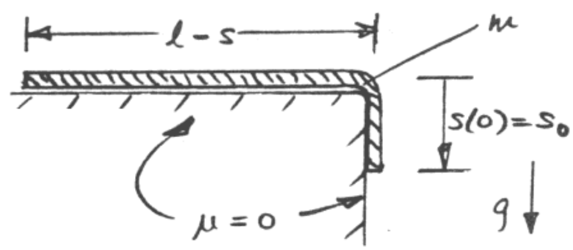
2.
Use the calculus of variations to obtain the function $\phi(\theta)$ describing the "great circle" path of minimum length on the surface of a sphere. This path connects spherical polar coordinates (θ_1, ϕ_1) with (θ_2, ϕ_2), in the general case where $\theta_1 \neq \theta_2$ and $\phi_1 \neq \phi_2$. Leave your answer in the form of an integral equation. [*Hint*: consider θ to be a "label" (like time t), and ϕ to be a coordinate (like $q(t)$).]

3.
Set up and solve the Euler-Lagrange equation for the Atwood machine, released from rest. (Two weights $m_1 < m_2$ are suspended via a massless string that is supported by a pulley in the form of a disk of radius R and mass M. The string moves without slipping on the pulley.)



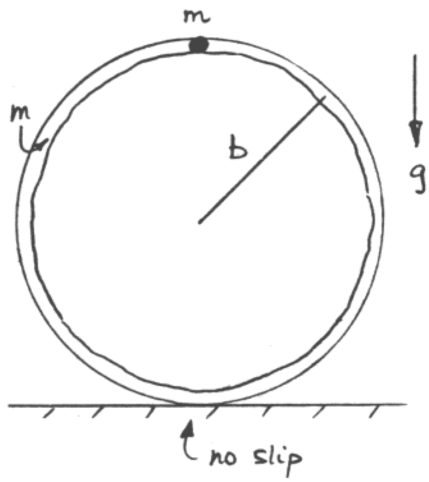
Use the height $y(t)$ of the smaller mass as the generalized coordinate.

4.
A chain of mass m and length l lies on a frictionless table. Initially the chain is at rest, with a length $s = s_0$ of the chain hanging off the table's end. This causes the chain to fall off the table. The part of the chain that remains on the table is straight, not coiled.



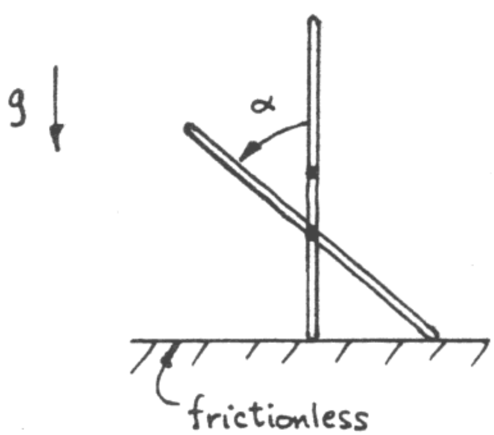
Using the Euler-Lagrange equation with s as the generalized coordinate, calculate the motion of the chain (before it falls off completely). Assume that the chain remains in contact with the corner and end of the table as shown (even though this in fact is true only for the early part of the motion).

5.
A bead of mass m moves inside a thin hoop-shaped pipe of average radius b, also of mass m. The pipe has a frictionless interior, so that the bead moves freely within the circumference of the hoop. But the coefficient of friction between the floor and the pipe's exterior is large, so that the hoop rolls on the floor without slipping.



The bead is released from rest at the top of the hoop. When the bead has fallen halfway to the floor, how far to the side will the hoop have moved?

6.
At $t = 0$, a thin uniform stick, resting on a frictionless floor, is erect and motionless. Let α represent the angle it makes with the vertical (initially $\alpha = 0$).



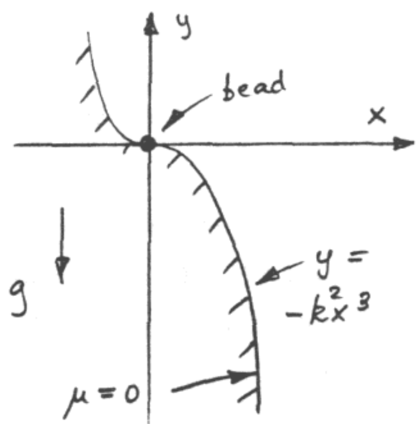
(**a**)
Use the Euler-Lagrange equation to obtain an equation relating $\ddot{\alpha}$ to α and $\dot{\alpha}$.
(**b**)
Because the floor is frictionless, total mechanical energy is conserved in this problem. Use this fact to relate $\dot{\alpha}$ to α.
(**c**)
Use the result of (**b**) to eliminate $\dot{\alpha}$ from your answer to (**a**), thereby obtaining an equation relating $\ddot{\alpha}$ to α alone. This equation should be valid for all values of α.
(**d**)
In the limit $\alpha \ll 1$, solve the result of (**c**) for the motion $\alpha(t)$.

7.
Continue to consider the stick in the previous problem. Use the method of Lagrange undetermined multipliers to find the force of constraint exerted by the floor on the stick, at the instant before the side of the stick impacts the floor.

8.
A bead moves under the influence of gravity on a frictionless surface described by

$$y = -k^2 x^3 ,$$

where k is a constant, and x and y are the horizontal and vertical coordinates.



The bead is released from rest at the origin. Use the method of Lagrange undetermined multipliers to solve for the coordinate $x = x_0$ at which it leaves the surface.